# FILM CONDENSATION ALONG AN INCLINED SURFACE IN A POROUS MEDIUM

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**Abstract-The** problems of steady film condensation outside a wedge or a cone embedded in a porous medium filled with a dry saturated vapor are investigated. As in classical film condensation problems, it is assumed that (a) the condensate and the vapor are separated by a distinct boundary with no two-phase zone in between, and (b) the condensate has constant properties. Within the boundary layer approximations, similarity solutions have been obtained for the temperature and flow fields in the condensate. Moreover, a closed form solution has been obtained for the Nusselt number which depends on the square root of the Rayleigh number and the dimensionless film thickness. The latter is found to be a function of a dimensionless parameter related to the degree of wall subcooling. Asymptotic cases for small and large wall subcoolings are also considered. As in the classical film condensation problems, it is found that the 'Nusselt'-type approximation (for small wall subcooling) overestimates the film thickness while underestimates the Nusselt number. An approximate expression for Nusselt number in terms of the degree of wall subcooling explicitly is also obtained.

#### **NOMENCLATURE**

- $C,$ specific heat of the fluid;
- f, dimensionless stream function;
- h. local heat transfer coefficient;
- $h_{f\boldsymbol{\varrho}},$ latent heat of vaporization;
- K, permeability of the porous medium;
- k. thermal conductivity of the saturated porous medium ;
- $\dot{m}$ mass flux across the interface;
- n,  $n = 0$  for wedge and  $n = 1$  for cone;
- $Nu_{x}$ local Nusselt number;
- $\mathbf{p}_{\star}$ pressure;
- $q,$ local heat transfer rate;
- r,  $r = x \sin \phi$ ;
- $Ra_{\infty}$ local Rayleigh number ;
- SC, dimensionless degree of wall subcooling;
- Sh, dimensionless degree of wall superheating;
- T, temperature;
- u. Darcy's velocity in x-direction ;
- $\boldsymbol{v}$ . Darcy's velocity in y-direction ;
- w, the width of the wedge;
- x, coordinate along the inclined surface;
- y, coordinate perpendicular to the inclined surface.

Greek symbols

- $\alpha$ , equivalent thermal diffusivity;<br> $\delta$ , boundary layer thickness;
- boundary layer thickness;
- $\eta$ , similarity variable;<br> $\theta$ , dimensionless temp
- dimensionless temperature;
- $\mu$ , viscosity of the fluid;<br> $\rho$ , density of the fluid;
- $\rho$ , density of the fluid;<br> $\phi$ , inclination angle;
- $\phi$ , inclination angle;<br> $\psi$ , stream function;
- $\psi$ , stream function;<br>  $\Gamma$ , total mass rate of
- total mass rate of condensate.

# Subscripts

- s, saturated condition;<br> $v$ , vapor phase;
- vapor phase;
- L, liquid phase ;
- $\infty$ , condition at infinity;
- w, condition at the wall.

# **INTRODUCTION**

THE PROBLEMS of two-phase flow in a porous medium involving phase change have important applications in geothermal energy utilization  $\lceil 1 \rceil$ , thermal enhancement of oil recovery [2] and *in situ* combustion processes, to name but three. When two-phase flow exists in a porous medium, it is known that Darcy's law is also applicable to both the liquid and the vapor phases provided that the concept of relative permeability be introduced. This is to account for the fact that the pore spaces are filled partly with vapor and partly with steam. However, because of the mathematical complexity of the governing equations, analytical solutions for two-phase flow in a porous medium involving phase change can be obtained only after simplifying assumptions have been made.

In this paper, the problems of steady film condensation along a wedge and a cone in a porous medium will be considered. The problem is formulated based on the standard approximations used in the classical film condensation problems  $\lceil 3-5 \rceil$ . It is assumed that (a) the condensate and the vapor are separated by a distinct boundary with no two-phase zone between; (b) the condensate has constant properties; and (c) condensate film is thin: such that boundary layer approximations are applicable. The first assumption was also employed by Parmentier [6] to study the problem of film boiling in a porous medium. Note that this approximation is akin to the so-called 'the abrupt interface approximation' in groundwater hydrology [7]. It is important to note that because of the first

approximation, the complexity of the relative permeability no longer exists in the problem, and that single-phase equations can be applied separately to the vapor and the condensate. As a result the mathematical problem of film condensation is considerably simplified, and similarity solutions for film condensation along a wedge and a cone are possible. Consequently, closed-form expressions of the Nusselt numbers are obtained in terms of the Rayleigh number and the dimensionless boundary layer thickness of the condensate. The latter is found to be a function of a dimensionless parameter related to the degree of wall. Asymptotic cases of a small and large wall subcooling are also considered. As in the classical film condensation problems, it is found that the 'Nusselt'-type approximation (for small wall subcooling) overestimates the film thickness while underestimates the Nusselt number.

#### **ANALYSIS**

Consider a two-dimensional wedge or a cone (having an included angle  $2\phi$ ) with wall temperature  $T_w$  is embedded in a porous medium filled with a dry saturated vapor at a saturated temperature  $T_s$  (corresponding to its pressure) as shown in Figs. l(a) and 1(b). If the wall temperature  $(T_w)$  is less than the saturated temperature  $(T_s)$ , a film of condensate will form adjacent to the surface and flows downward because of gravity. To investigate the problem of film condensation about an inclined surface in a porous medium, the following assumptions will be made :

- (1) The condensateand the-dry saturated vapor are separated by a distinct interface at  $y = \delta_L$ .
- (2) The condensate film is thin such that boundary layer approximations are applicable.
- (3) The properties of the porous medium, the dry saturated vapor, and the condensate are constant.
- (4) The inclination angle  $\phi$  (with respect to the vertical) is small such that the component of the

gravitational force normal to the surface is negligible.

- (5) The saturated temperature  $(T<sub>s</sub>)$  is a constant
- (6) Darcy's law is applicable to both the dry vapor and liquid phases in the porous medium.

Assumptions  $(1)$ - $(5)$  are the usual approximations used in the classical film condensation problems. Under the aforementioned assumptions, the governing equations for the condensate at  $y < \delta_L$  are

$$
\frac{\partial}{\partial x}(r^n u_L) + \frac{\partial}{\partial y}(r^n v_L) = 0 \tag{1}
$$

$$
u_L = -\frac{K}{\mu_L} (\rho_v - \rho_L) g \cos \phi \qquad (2)
$$

$$
u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial y} = \alpha_L \frac{\partial^2 T_L}{\partial y^2}
$$
 (3)

where the subscripts  $L$  and  $v$  denote the quantities associated with the liquid **and** vapor phases ; u and v are the Darcy's velocities in the x- and y-directions;  $\rho$ and  $\mu$  are the density and viscosity of the condensate;  $K$  and  $\alpha$  are the permeability and the equivalent thermal diffusivity of the porous medium;  $p$  and  $T$  are the pressure and temperature. In equation (1)  $r =$ x sin  $\phi$  and  $n = 0$  for a wedge and  $n = 1$  for a cone. Equation (2) shows that the condensate is flowing downward along the impermeable surface with a constant velocity, since both  $\rho_L$  and  $\rho_v$  are assumed to be constant.

The boundary conditions at the wall are

 $\dot{n}$ 

$$
y = 0
$$
,  $v_L = 0$  and  $T_L = T_w$ . (4a, b)

At the liquid-vapor interface at  $y = \delta_L$ , the continuity of temperature mass flux and energy flux are

$$
y = \delta_L, \quad T_L = T_s \tag{5}
$$

$$
\dot{m} = \rho_L \left( u_L \frac{d \delta_L}{dx} - v_L \right)_{y = \delta_L} \tag{6}
$$

$$
ih_{fg} = -k_{m,L} \left(\frac{\partial T_L}{\partial y}\right)_{y=\delta_L} \tag{7}
$$



FIG. 1. Coordinate systems for film condensation along a wedge ( $n = 0$ ) and a cone ( $n = 1$ ).

where  $\dot{m}$  is the mass flux of condensate across the subcooling; and interface,  $h_{fa}$  is the latent heat at  $T_x$ , and  $k_{m,L}$  is the thermal conductivity of the porous medium saturated<br>with liquid. Substituting equation (6) into (7) gives the which is the dimensionless liquid film thickness. with liquid. Substituting equation  $(6)$  into  $(7)$  gives the following interface condition Equation (15) with boundary condition (17a) can be

$$
h_{f\theta}\rho_L\bigg(u_L\frac{\mathrm{d}\delta_L}{\mathrm{d}x}-v_L\bigg)_{y=\delta_L}=-k_{m,L}\bigg(\frac{\partial T_L}{\partial y}\bigg)_{y=\delta_L}.\quad (8)
$$

sate such that  $\theta''_L + (n + \frac{1}{2})\eta_L\theta'_L = 0$  (22)

$$
u_L = \frac{1}{r^n} \frac{\partial \psi_L}{\partial y} \quad \text{and} \quad v_L = -\frac{1}{r^n} \frac{\partial \psi_L}{\partial x} \tag{9}
$$

so that the continuity equation is satisfied automatically. In terms of the stream function, equations (2) and Equation (22) with boundary conditions (17b) and (18) (3) with boundary conditions (4a) and (8) become has the following exact solution  $(3)$  with boundary conditions  $(4a)$  and  $(8)$  become

$$
\frac{1}{r^n} \frac{\partial \psi_L}{\partial y} = \frac{K}{\mu_L} (\rho_L - \rho_v) g \cos \phi \tag{10}
$$

$$
\frac{1}{r^n} \left[ \frac{\partial \psi_L}{\partial y} \frac{\partial T_L}{\partial x} - \frac{\partial \psi_L}{\partial x} \frac{\partial T_L}{\partial y} \right] = \alpha_L \frac{\partial^2 T_L}{\partial y^2}
$$
 (11) and consequently  

$$
\theta'_L(\eta_L) = -\frac{\sqrt{(2n+1)}}{L \exp[(2n+1)n^2/4] \text{erff}}
$$

and

$$
y = 0, \quad \frac{\partial \psi_L}{\partial x} = 0 \tag{12}
$$

$$
\frac{h_{fg}\rho_L}{r^n} \left[ \frac{\partial \psi_L}{\partial y} \frac{d\delta_L}{dx} + \frac{\partial \psi_L}{\partial x} \right]_{y=\delta_L}
$$
\n
$$
= -k_{m,L} \left( \frac{\partial T_L}{\partial y} \right)_{y=\delta_L}
$$
\n
$$
= -k_{m,L} \left( \frac{\partial T_L}{\partial y} \right)_{y=\delta_L}
$$
\n(13) which is obtained by substitu equation (23). For the special  
parallel flat plate (n = 1 and

Equations (10) and (11) with boundary conditions  $(4b)$ ,  $(5)$ ,  $(12)$  and  $(13)$  can now be solved by similarity transformations. To this end, we introduce the following similarity variables

$$
\eta_L = \sqrt{(Ra_{x,L})y/x} \tag{14a}
$$

$$
\psi_L = r^n \alpha_L \sqrt{(Ra_{x,L}) f_L(n_L)} \tag{14b}
$$

$$
\theta_L(\eta_L) = \frac{T_L - T_s}{T_w - T_s} \tag{14c}
$$

Rayleigh number of the condensate. In terms of the similarity variables, equations (10) and (11) with approximations applied to the Darcy's law, one boundary conditions  $(4b)$ ,  $(5)$ ,  $(12)$  and  $(13)$  are obtains

$$
f'_L = 1\tag{15}
$$

$$
\theta_L'' + (n + \frac{1}{2}) f_L \theta_L' = 0 \tag{16}
$$

subject to the boundary conditions at the wall

$$
f_L(0) = 0, \quad \theta_L(0) = 1 \tag{17a,b}
$$

and the interface conditions

$$
\theta_L(\eta_{L\delta}) = 0 \tag{18}
$$

$$
Sc \theta_L'(\eta_{L\delta}) = -(n + \frac{1}{2}) f_L(\eta_{L\delta}) \qquad (19)
$$

 $c_{PL}(T_s - T_w)/h_{fg}$  is a measure of degree of wall or

$$
\eta_{L\delta} = (\eta_L)_{y = \delta_L} = \sqrt{(Ra_{x,L})\delta_L/x} \tag{20}
$$

integrated to give

$$
f_L = \eta_L. \tag{21}
$$

We now introduce the stream function of the conden-<br>Substituting equation (21) into (16) and (19) yields

$$
\theta_L'' + (n + \frac{1}{2})\eta_L \theta_L' = 0 \tag{22}
$$

and

$$
Sc\theta_L'(\eta_{L\delta}) = -(n+\tfrac{1}{2})\eta_{L\delta}.
$$
 (23)

$$
\frac{1}{r^n} \frac{\partial \psi_L}{\partial y} = \frac{K}{\mu_L} (\rho_L - \rho_v) g \cos \phi
$$
\n
$$
(10) \qquad \theta_L(\eta_L) = 1 - \frac{\text{erf}[(2n+1)^{1/2} \eta_L/2]}{\text{erf}[(2n+1)^{1/2} \eta_L/2]} \qquad (24a)
$$

and consequently

$$
\theta'_{L}(\eta_{L}) = -\frac{\sqrt{(2n+1)}}{\sqrt{\pi \exp[(2n+1)\eta_{L}^{2}/4] \operatorname{erf}[(2n+1)^{1/2} \eta_{L\delta}/2]}}
$$
\n(24b)

where  $\eta_{L\delta}$  is determined from

$$
\frac{\partial \varphi_L}{\partial r} \left[ \frac{\partial \psi_L}{\partial y} \frac{d \delta_L}{dx} + \frac{\partial \psi_L}{\partial x} \right]_{v = \delta_L} \qquad \qquad \text{Sc} = \sqrt{\pi} \left[ (2n+1)^{1/2} \eta_{L\delta} / 2 \right] \times \exp \left[ (2n+1) \eta_{L\delta}^2 / 4 \right] \text{erf} \left[ (2n+1)^{1/2} \eta_{L\delta} / 2 \right] \tag{25}
$$

which is obtained by substituting equation (24b) into equation (23). For the special case of an isothermal vertical flat plate ( $n = 1$  and  $\phi = 0$ ), equations (24a) and (25) are similar to the solution obtained by Parmentier [6] for film boiling along a heated vertical plate in a porous medium filled with a saturated vapor, provided that the roles of the vapor and the liquid layer be interchanged and *Sc* be replaced by *Sh* (where  $Sh \equiv$  $c_{pv}(T_w - T_s)/h_{fg}$ ). It is not surprising to note that film condensation and film boiling in a porous medium has much in common.

We now consider the flow field in the vapor phase at  $y > \delta_L$ . Since the vapor is at constant temperature  $T_s$ , where  $Ra_{x,L} = K(\rho_L - \rho_v) g \cos \phi x / \mu_L \alpha_L$  is the local the energy equation is automatically satisfied. With the Rayleigh number of the condensate. In terms of the aid of the continuity equation and the boundary layer

$$
f'_{L} = 1
$$
 (15)  $u_{v} = 0$  and  $v_{v} = f(x)$ . (26a, b)

From the interface mass continuity equation

$$
\dot{m} = \rho_v \left( u_v \frac{d\delta_v}{dx} - v_v \right)_{y = \delta_L}
$$

$$
= \rho_L \left( u_L \frac{d\delta_L}{dx} - v_L \right)_{y = \delta_L} \quad (27)
$$

and with the aid of equations (14b), one obtains

$$
\text{Sc } \theta_L'(\eta_{L\delta}) = -(n+1)L(\eta_{L\delta}) \qquad (19)
$$
\n
$$
\text{where the primes in equations (15), (16) and (19)} \qquad \dot{m} = (n+\frac{1}{2})\rho_L \left[ \frac{K\alpha_L(\rho_L - \rho_v)g\cos\phi}{\mu_L x} \right]^{1/2} \eta_{L\delta} \text{ (28a)}
$$
\n
$$
\text{denote differentiation with respect to } \eta_L; \qquad \text{Sc} =
$$

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FIG. 2. Dimensionless film thickness vs dimensionless degree of wall subcooling.

$$
\dot{m} = (n + \frac{1}{2}) \frac{\rho_L K (\rho_L - \rho_v) g \cos \phi}{\mu_L} \frac{\delta}{x}
$$
 (28b)

where we have made use of equation (20). It follows from equations (29b), (28a) and (27) that

$$
v_v = -\frac{(n+\frac{1}{2})\rho_L}{\rho_\infty} \times \left[\frac{K\alpha_L(\rho_L-\rho_v)g\cos\phi}{\mu_L x}\right]^{1/2} \eta_{L\delta} \quad (29)
$$

which shows that the vapor is moving toward the interface. The total mass rate of condensate along a wedge and a cone can be computed according to

$$
\Gamma = \int_0^x (2\pi r)^n W^{1-n} m \, dx = W^{1-n} (2\pi \sin \phi)^n \int_0^x x^n m \, dx \tag{30}
$$

where *W* is the width of the wedge. Substituting equation (28a) into (30), performing the integration and making use of equation (20), one obtains

$$
\Gamma = \frac{\rho_L K (\rho_L - \rho_v) g \cos \phi \delta_L W^{1-n} (2\pi r)^n}{\mu_L}.
$$
 (31)

Equation (31) could have been obtained directly from the mass consideration in the liquid phase, i.e.  $\Gamma$  =  $\rho_L u_L \delta_L W^{1-n}(2\pi r)^n$  where  $u_L$  is given by equation (2).

## NUMERICAL RESULTS AND DISCUSSION

Equation (25) is plotted as a solid line in Fig. 2 where it is shown that the dimensionless thickness of the liquid film increases as  $Sc$  is increased. With the aid of



FIG. 3. Dimensionless temperature and velocity profiles in the condensate.

Fig. 2, the dimensionless temperature profiles of the condensate given by equation (24a) can now be plotted as a function of SC, as shown in Fig. 3. The dimensionless velocity in the direction along the impermeable surface is plotted as a horizontal line in the same plot. From Figs. 2 and 3 it can be deduced that the boundary layer thickness of a cone is smaller by a factor of  $1/\sqrt{3}$  than that of a wedge.

The surface heat flux along the impermeable surface is

$$
q_w = -k_{m,L} \left(\frac{\partial T_L}{\partial y}\right)_{y=0}
$$
  
= 
$$
\frac{k_{m,L}(T_w - T_s) \sqrt{(Ra_x, L)}}{x} \left[-\theta'_L(0)\right]
$$
 (32)

where  $k_{m,L}$  is the thermal conductivity of the porous medium saturated with liquid. Substituting equation (24b) into (32) yields

$$
q_{\mathbf{w}} = \left\{ \frac{(2n+1) k_{\mathbf{m},L}(T_{\mathbf{w}} - T_{\mathbf{s}}) \sqrt{(Ra_{\mathbf{x},L})}}{\pi} \right\}^{1/2}.
$$
 (33)

The local Nusselt number is defined as

$$
Nu_x \equiv \frac{hx}{k_{m,L}} = \frac{q_w x}{k_{m,L}(T_w - T_s)}.
$$
 (34a)

Substituting equation (33) into (34a), one obtains

$$
\frac{Nu_{x}}{\sqrt{[(2n+1)Ra_{x,L}]}} = \frac{1}{\sqrt{\pi \,\text{erf}[(2n+1)^{1/2}\eta_{Ld}/2]}}.(34b)
$$

Equation (34b) is an exact solution for Nusselt number in terms of the dimensionless boundary layer thickness, which is implicitly a function of  $Sc$  through equation (25). It will be convenient to obtain an expression for Nusselt number in terms of the wall subcooling explicitly. To this end, we now consider equations (24), (34b) and (23) for the limiting cases of  $\eta_{L\delta} \to 0$  and  $\eta_{L\delta} \to \infty$ :

(1) As  $\eta_{L\delta} \rightarrow 0$ , equations (24) can be expanded for small  $\eta_{L\delta}$  to give

$$
\theta_L = 1 - \frac{\eta_L}{\eta_{L\delta}} \tag{35}
$$

$$
\theta'_{L} = -\frac{1}{\eta_{L\delta}}\tag{35}
$$

and consequently

$$
\eta_{L\delta}^2 = \frac{2Sc}{(2n+1)}\tag{37}
$$

which is obtained by substituting equation (36) into (23). Equation (37) shows that  $Sc \rightarrow 0$  as  $\eta_{L\delta} \rightarrow 0$ . Substituting (37) into (35) gives

$$
\theta_L(\eta_L) = 1 - \frac{\eta_L \sqrt{(2n+1)}}{\sqrt{(2Sc)}}\tag{38}
$$

which shows that the temperature of the condensate is a linear function of distance. Similarly, equation (34) can be expanded for small  $\eta_{L\delta}$  to give

$$
\frac{Nu_{x}\sqrt{(2Sc)}}{\sqrt{[(2n+1)Ra_{x,L}]}} = 1
$$
\n(39)

where we have made use of equation (37).

With the aid of equations (20) and (37), the film thickness for this limiting case is given by

$$
\frac{\delta_L}{x} = \left[ \frac{2Sc}{(2n+1)Ra_{x,L}} \right]^{1/2}.
$$
 (40)

It should be noted that equations  $(38)$ - $(40)$  are similar to the 'Nusselt'-type of analysis in the classical film condensation problems [5], and can therefore be obtained from a simple control volume analysis by writing mass and energy balance with the aid of the Darcy's law and the assumption of a linear temperature profile inside the liquid film.\* Equations (37) and (39) are plotted as dashed lines in Figs. 2 and 4. As in the classical film condensation problems, it is shown in these figures that the 'Nusselt'-type approximation overestimates the boundary layer thickness while underestimates the Nusselt number.

(2) As  $\eta_{L\delta} \rightarrow \infty$ , equation (24) and (34) can be expanded for large  $\eta_{L\delta}$  to give

$$
\theta_L = 1 - \text{erf}[(2n+1)^{1/2}\eta_L/2] \tag{41}
$$

$$
\frac{Nu_x}{\sqrt{[(2n+1)Ra_{x,L}]}} = 0.5642. \tag{42}
$$

and

$$
Sc = \sqrt{\pi} \left[ \frac{\sqrt{(2n+1)}}{2} \eta_{L\delta} \right] \exp \left[ \frac{(2n+1)}{4} \eta_{L\delta}^2 \right]. \tag{43}
$$

Equation (42) is plotted as horizontal dashed lines on the right-hand margin of Fig. 4. Equation (43) implies that  $Sc \to \infty$  as  $\eta_{L\delta} \to \infty$ . Note that for this limiting case the boundary layer approximations are accurate only if  $Ra_{x,L} \rightarrow \infty$  so that  $\delta/x < 1$ .

Equations (39) and (42) suggest that it is convenient to plot

$$
\frac{Nu_x\sqrt{2Sc}}{\sqrt{[(2n+1)Ra_{x.L}]}}
$$
 vs  $\sqrt{2Sc}$ ,

which can be obtained by first assuming a value of  $\eta_{L\delta}$ and computing Sc and  $Nu_x$  from equations (25) and (34b), respectively. The results of the computations are plotted as a solid line in Fig. 5. The straight (dashed) lines in Fig. 5 are the asymptotic limits of small and

<sup>\*</sup> After this work had been completed, the author received an unpublished paper [8] from Dr H. Hardee who had also considered the problem of film condensation along a vertical cold plate in a porous medium and obtained an approximate solution similar to equations (39) and (40) (with  $n = 0$  and  $\phi$  $= 0$ ) independently by an integral method.

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FIG. 4. Heat transfer results for film condensation.

large wall subcoolings given by equations (39) and (42). An approximate expression for Nusselt number valid for the whole range of wall subcooling can now be constructed by the method described by Churchill and Usag [9]. To this end we write the Nusselt number in the following form

$$
\frac{Nu_x}{\sqrt{[(2n+1)Ra_{x,L}]}} = \left[\left(\frac{1}{\sqrt{2Sc}}\right)^m + \left(\frac{1}{\sqrt{\pi}}\right)^m\right]^{1/m} (44)
$$

which would reduce to the asymptotic expressions

given by equations (39) and (42) for small and large wall subcoolings, respectively. The value of  $m$  in equation (44) is then determined by comparing the right-hand side of equation (44) to the right-hand side of equation (34b) at different values of Sc. It was found that if the value of  $m = 2$  is chosen in equation (44), the right-hand side of equation (44) is closest to the exact solution given by the right-hand side of equation (34b). Equation (44) with  $m = 2$  gives



FIG. 5. The quantity  $\{Nu_x/\sqrt{[(2n + 1)Ra_{x,L}]\}}\sqrt{2Sc}$  vs  $\sqrt{2Sc}$ .

$$
\frac{Nu_{x}\sqrt{2Sc}}{\sqrt{[(2n+1)Ra_{x,L}]}} = \left[1 + \frac{2Sc}{\pi}\right]^{1/2}
$$
 (45)

which is an approximate expression for Nusselt number that is accurate to the fourth significant figures (as compared to the exact solution) for the whole range of SC.

# CONCLUDING REMARKS

The problems of film condensation along a wedge and a cone in a porous medium have been investigated based on the standard approximations used in the classical film condensation literature. Although the assumption of no mixed zone existing between the film and the saturation vapor is an accurate one for the classical film condensation problems, the validity of this assumption for film condensation in a porous medium must await experimental confirmation. As noted earlier, the governing equations and boundary conditions (under the assumptions stated) for the present problem are identical to those of film boiling in a porous medium filled with a saturated liquid, provided that the roles of the vapor and the liquid are being interchanged. Thus, Figs. 2-5 will also apply to the problems of film boiling about a wedge and a cone in a porous medium if the subscript  $L$  is replaced by  $v$ , and  $Sc$  is replaced by  $Sh$  (a dimensionless parameter related to the degree of wall superheating).

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#### CONDENSATION EN FILM LE LONG D'UNE SURFACE INCLINEE DANS UN MILIEU POREUX

Résumé—On étudie le problème de la condensation en film le long d'un dièdre ou d'un cone dans un milieu poreux empli de vapeur saturée sèche. Comme dans les problèmes classiques de condensation en film, on suppose que (i) le condensat et la vapeur sont séparés 'ar une frontière sans zone diphasique, et (ii) le condensat a des propridtes constantes. Dans les approximations de la couche limite, on obtient des solutions de similitude pour la temperature et la vitesse dans le condensat. Une solution analytique est obtenue pour le nombre de Nusselt qui dépend de la racine carrée du nombre de Rayleigh et de l'épaisseur adimensionnelle du film. Cette demibe est une fonction du parambtre adimensionnel lie au sous-refroidissement de la paroi. Des cas asymptotiques pour des petits et des grands sous-refroidissements de la paroi sont aussi considérés. Comme dans les problemes classiques de condensation en film, on trouve que l'approximation de type "Nusselt" (pour un faible sous-refroidissament de la paroi) surestime l'épaisseur du film et sous-estime le nombre de Nusselt. On obtient une expression approchée du nombre de Nusselt en fonction du degré de sous-refroidissement de la paroi.

#### FILMKONDENSATION AN EINER GENEIGTEN OBERFLÄCHE IN EINEM PORÖSEN MEDIUM

Zusammenfassung-In diesem Aufsatz werden die Probleme der stetigen Filmkondensation an Keilen und Kegeln in einem porosen Medium untersucht, das mit trocken gesattigtem Dampf gefiillt ist. Wie bei den klassischen Filmkondensationsproblemen wird angenommen, dal3 erstens Kondensat und Dampf durch eine definierte Grenzfläche voneinander getrennt sind, ohne daß sich dazwischen eine Zweiphasenzone befindet und daß zweitens das Kondensat konstante Stoffwerte besitzt. Mit Hilfe der Grenzschicht-Näherungen erhält man Ähnlichkeits-Lösungen für das Temperatur- und Geschwindigkeitsfeld im Kondensat. Darüber hinaus gelang es, eine geschlossene Lösung für die Nusselt-Zahl zu entwickeln; diese hängt vom Quadrat der Rayleigh-Zahl und von der dimensionslosen Filmdicke ab. Letztere ist eine Funktion eines dimensionslosen Parameters, der seinerseits mit dem Grad der Wandunterkiihlung verkniipft ist. Es werden such Grenzfiille fiir kleine und grol3e Wandunterkiihlungen betrachtet. Wie bei den klassischen Problemen der Filmkondensation ergibt sich, daB die 'Nusselt-Approximation' (fur kleine Wandunterkühlungen) die Filmdicke zu groß und die Nusselt-Zahl zu klein liefert. Man erhält explizit einen Näherungsausdruck für die Nusselt-Zahl in Abhängigkeit vom Grad der Wandunterkühlung.

# ПЛЕНОЧНАЯ КОНДЕНСАЦИЯ НА НАКЛОННОЙ ПОВЕРХНОСТИ В ПОРИСТОЙ СРЕДЕ

Аннотация - Исследуется стационарная пленочная конденсация на поверхности клина или конуса, помещенного в пористую среду, заполненную сухим насыщенным паром. Аналогично классическим задачам по пленочной конденсации предполагается, что (1) между конденсатом и паром имеется четкая граница и отсутствует двухфазная зона и что (2) конденсат характеризуется постоянными свойствами. В приближении пограничного слоя получены решения для температурных и динамических полей в конденсате. Кроме того получено выражение для числа Нуссельта, пропорционального корню квадратному из числа Релея и безразмерной толщине пленки. Найдено, что последняя зависит от безразмерного параметра, связанного со степенью недогрева на стенке. Рассмотрены также асимптотические случаи больших и малых недогревов на стенке. Аналогично классическим задачам по пленочной конденсации найдено, что зависимость типа «приближения Нуссельта» (для небольших недогревов на стенке) дает завышенные значения толщины пленки и заниженные значения числа Нуссельта. Для последнего получена приближенная зависимость, выраженная через степень недогрева на стенке.